Instructions: Complete each of the following exercises for practice.

1. Compute the length of the give curve.

   (a) $\mathbf{r}(t) = \langle t, 3\cos(t), 3\sin(t)\rangle$, $-5 \le t \le 5$

   (b) $\mathbf{r}(t) = \left\langle 2t, t^2, \frac{1}{3}t^3\right\rangle$, $0 \le t \le 1$

   (c) $\mathbf{r}(t) = \sqrt{2}t\mathbf{i} + e^t\mathbf{j} + e^{-t}\mathbf{k}$, $0 \le t \le 1$

   (d) $\mathbf{r}(t) = \cos(t)\mathbf{i} + \sin(t)\mathbf{j} + \ln(\cos(t))\mathbf{k}$, $0 \le t \le \frac{\pi}{4}$

   (e) $\mathbf{r}(t) = \mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$, $0 \le t \le 1$

   (f) $\mathbf{r}(t) = t^2\mathbf{i} + 9t\mathbf{j} + 4t^{\frac{3}{2}}\mathbf{k}$, $1 \le t \le 4$

2. Parameterize the curve $\mathbf{r}(t)$ with respect to arc length measured from P in the direction of increasing t.

   (a) $\mathbf{r}(t) = \langle 5 - t, 4t - 3, 3t\rangle$, $P = (4, 1, 3)$

   (b) $\mathbf{r}(t) = \langle e^t\sin(t), e^t\cos(t), \sqrt{2}e^t\rangle$, $P = (0, 1, \sqrt{2})$

3. Reparametrize the curve $\mathbf{r}(t) = \dfrac{1-t^2}{1+t^2}\mathbf{i} + \dfrac{2t}{t^2+1}\mathbf{j}$ with respect to arc length measured from the point $(1, 0)$ in direction of increasing t; simplify where possible. What can you conclude about the curve?

4. Compute unit tangent and unit normal vectors to $\mathbf{r}(t) = \langle t, 3\cos(t), 3\sin(t)\rangle$; compute the curvature $\kappa(t)$.

5. Compute the curvature $\kappa(t)$ for the curve $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + e^t\mathbf{k}$.

6. Compute the curvature $\kappa(t)$ for $\mathbf{r}(t) = \langle t^2, \ln(t), t\ln(t)\rangle$; what is the curvature at the point $P = (1, 0, 0)$?

7. Compute the unit tangent, unit normal, and binormal vectors of $\mathbf{r}(t) = \langle\cos(t), \sin(t), \ln(\cos(t))\rangle$.

8. Compute equations of the normal and osculating planes of the given curve at the given point.

   (a) $x = \sin(2t)$, $y = -\cos(2t)$, $z = 4t$ at $(0, 1, 4\pi)$

   (b) $x = \ln(t)$, $y = 2t$, $z = t^2$ at $(0, 2, 1)$

9. Prove that $\dfrac{\partial\mathbf{T}}{\partial s} = \kappa\mathbf{N}$ for all curves $\mathbf{r}(t)$.